HYDRO STATICS

RELATIVE EQUILIBRIUM OF LIQUIDS

Relative Equilibrium of Liquids

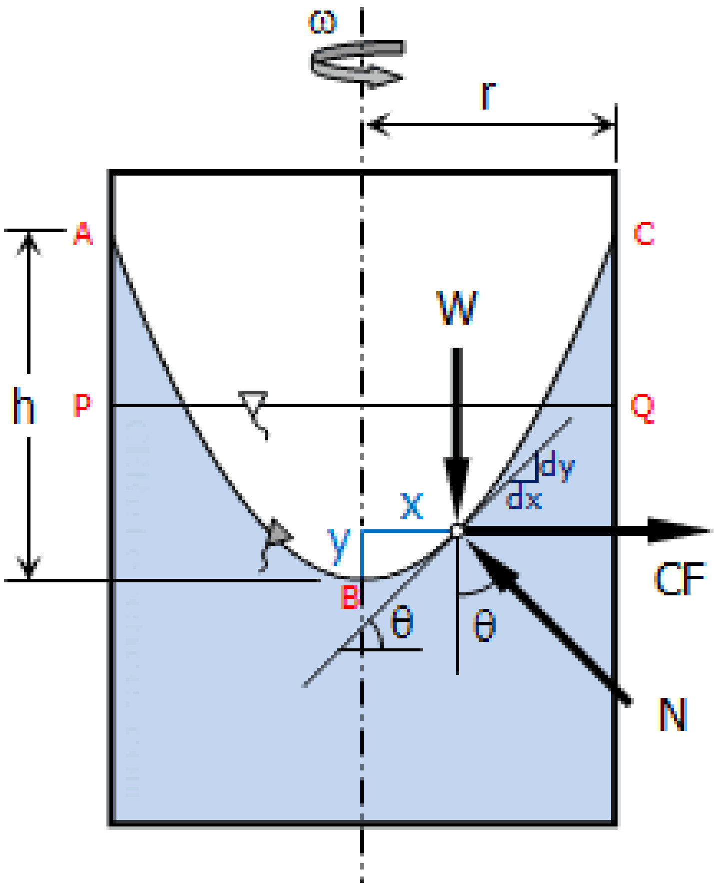
Introduction

Relative equilibrium of liquid is a condition where the whole mass of liquid including the vessel in which the liquid is contained, is moving at uniform accelerated motion with respect to the earth, but every particle of liquid have no relative motion between each other. There are two cases of relative equilibrium that will be discussed in this section: linear translation and rotation. Note that if a mass of liquid is moving with constant speed, the conditions are the same as static liquid in the previous sections.

Relative Equilibrium of Liquids

Rotation (Rotating Vessel)

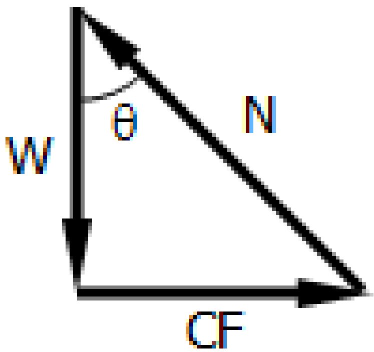
When at rest, the surface of mass of liquid is horizontal at PQ as shown in the figure. When this mass of liquid is rotated about a vertical axis at constant angular velocity ω (*in radians per second*), it will assume the surface ABC which is parabolic. Every particle is subjected to a centrifugal force (or reversed normal effective force) $CF = m\omega^2 x$ which produces centripetal acceleration towards the center of rotation. Other forces that acts are gravity force $W = mg$ and normal force N.



Relative Equilibrium of Liquids

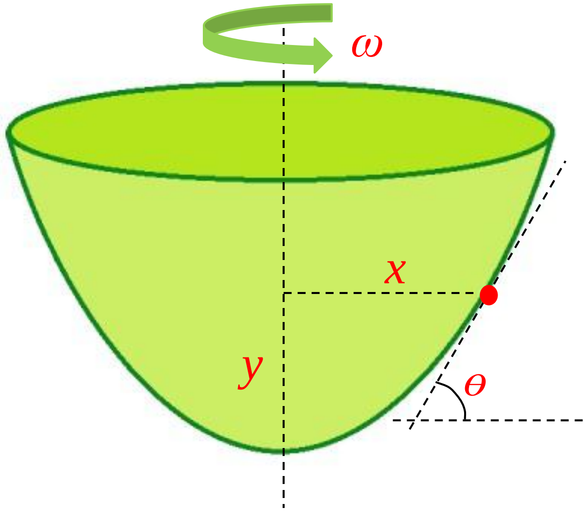
Rotation (Rotating Vessel)

From the force polygon,



$$\tan\theta = \frac{CF}{W}$$

$$\tan\theta = \frac{m\omega^2 x}{mg}$$



$$\tan\theta = \frac{\omega^2 x}{g}$$

Where tan θ is the slope at the surface of paraboloid at any distance x from the axis of rotation.

Relative Equilibrium of Liquids

Rotation (Rotating Vessel)

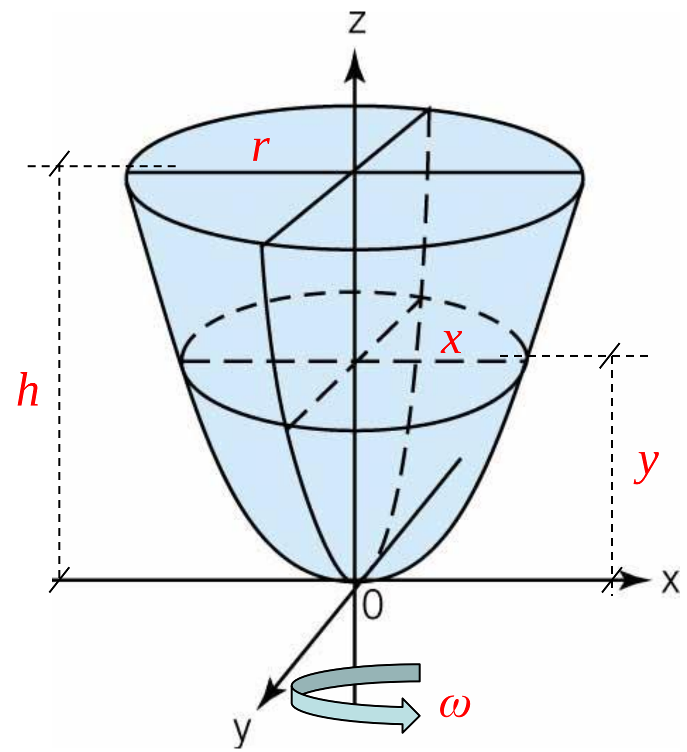
From Calculus, y' = slope, thus

$$\frac{dy}{dx} = \tan\theta$$

$$\frac{dy}{dx} = \frac{\omega^2 x}{g}$$

$$dy = \frac{\omega^2}{g} x \, dx$$

$$\int dy = \frac{\omega^2}{g} \int x \, dx$$



$$y = \frac{\omega^2 x^2}{2g}$$

Relative Equilibrium of Liquids

Rotation (Rotating Vessel)

For cylindrical vessel of radius r revolved about its vertical axis, the height h of paraboloid is

$$h = \frac{\omega^2 r^2}{2g}$$

Other Formulas

By squared-property of parabola, the relationship of y, x, h and r is defined by

$$\frac{r^2}{h} = \frac{x^2}{y}$$

Relative Equilibrium of Liquids

Rotation (Rotating Vessel)

Other Formulas

Volume of paraboloid of revolution

$$V = \frac{1}{2}\pi r^2 h$$

Important conversion factor

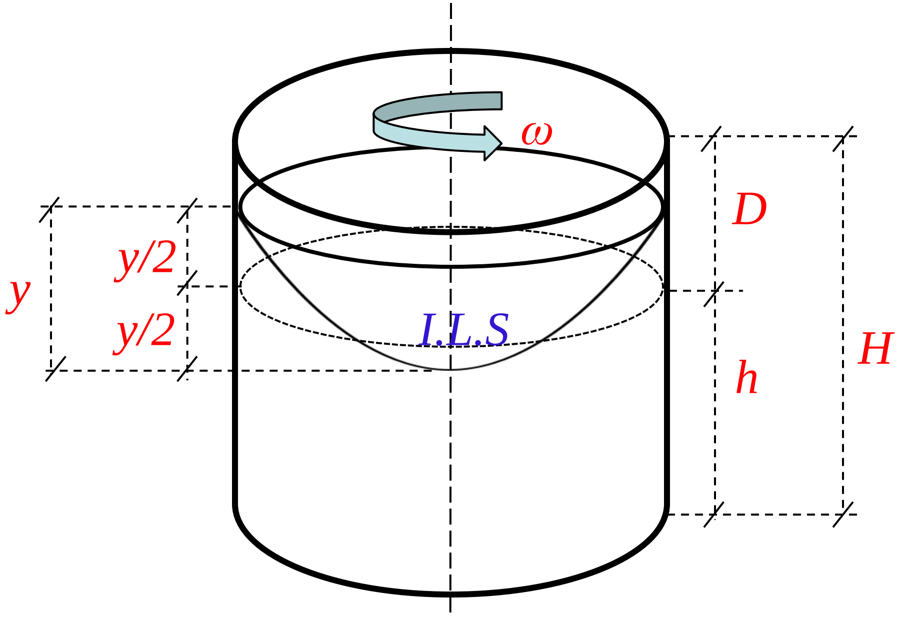
$$1 \text{ rpm} = \frac{1}{30}\pi \text{ rad/sec}$$

Relative Equilibrium of Liquids

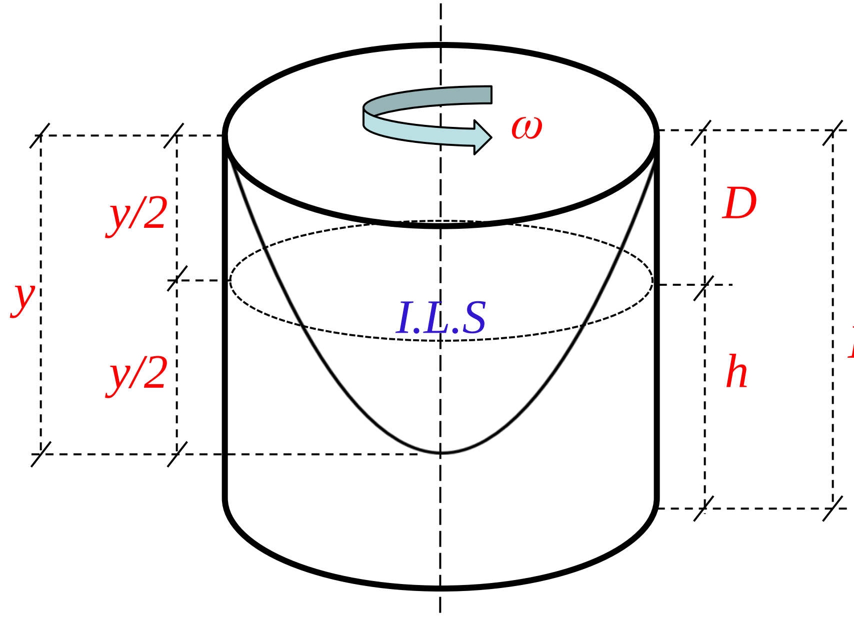
Rotation (Rotating Vessel)

Liquid Surface Conditions

For open cylindrical containers more than half-full of liquid, rotated about its vertical axis ($h > H/2$):



$y/2 < D$
(No liquid spilled)

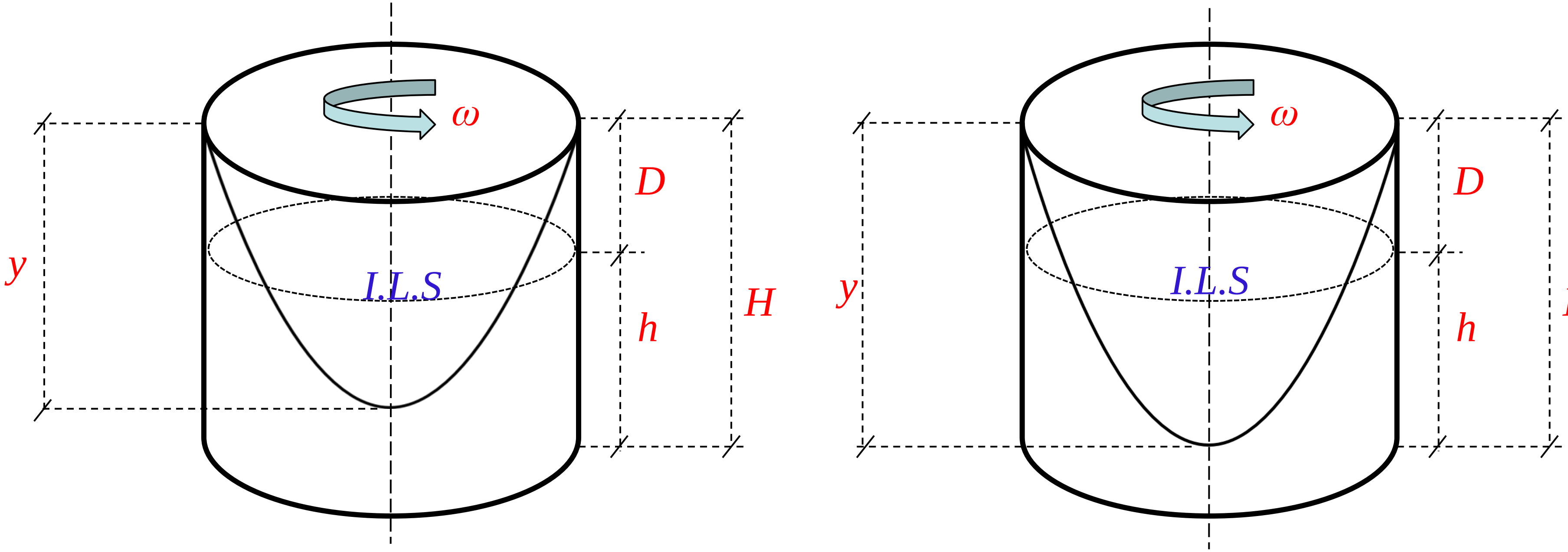


$y/2 = D$
(Liquid surface just touching the top rim, no liquid spilled)

Relative Equilibrium of Liquids

Rotation (Rotating Vessel)

Open cylindrical containers ($h > H/2$):



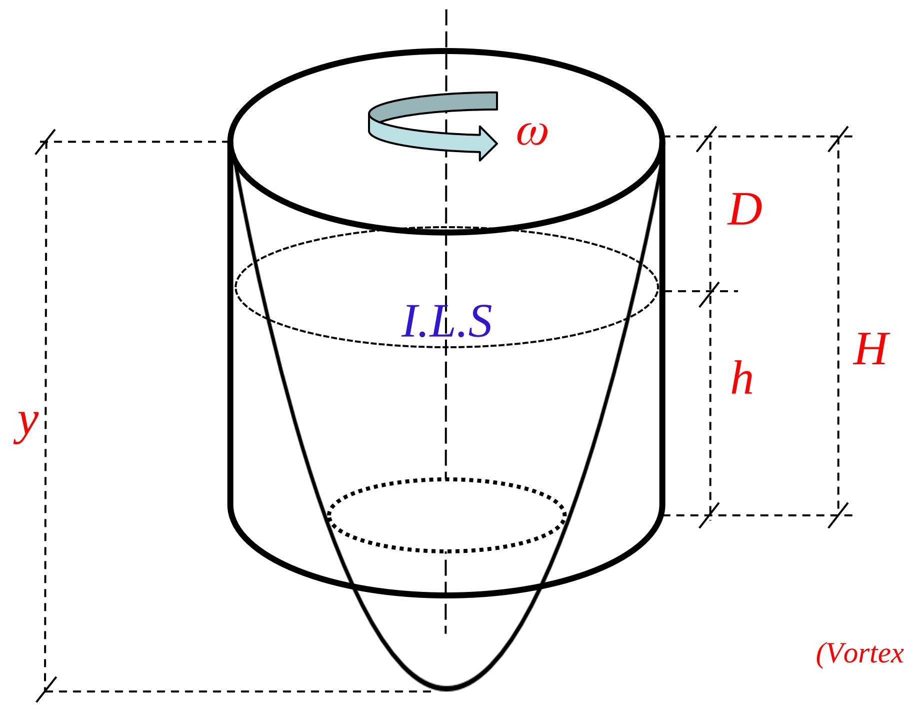
$y/2 > D$
(Some liquid spilled)

$y = H$
(Vortex at the bottom, some liquid spilled)

Relative Equilibrium of Liquids

Rotation (Rotating Vessel)

Open cylindrical containers ($h > H/2$):



y > H

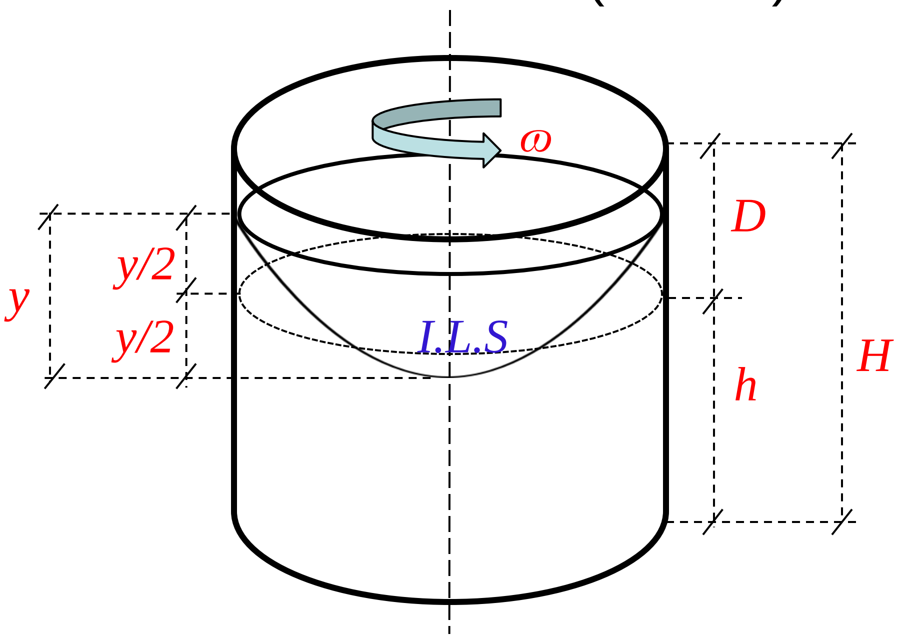
(Vortex (imaginary) below the bottom, some liquid spilled)

Relative Equilibrium of Liquids

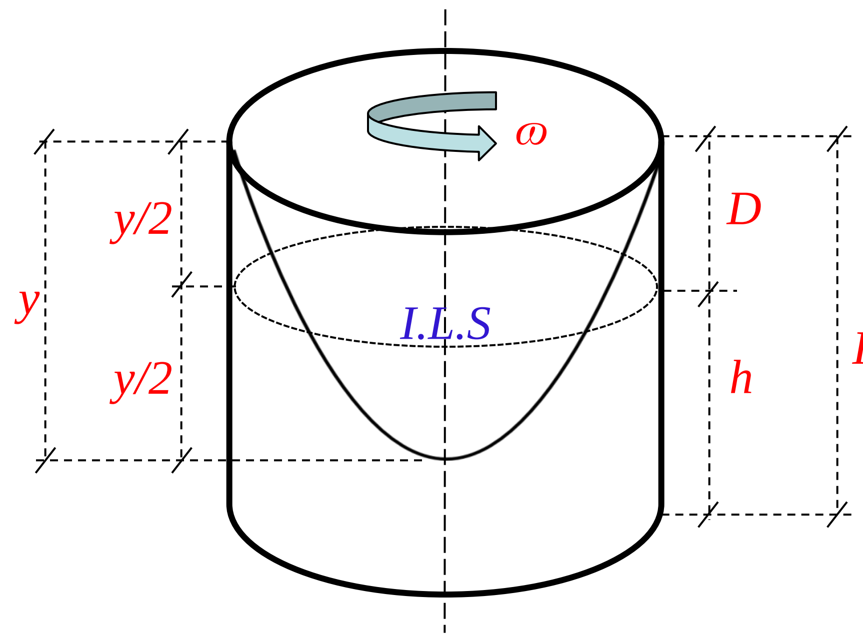
Rotation (Rotating Vessel)

Liquid Surface Conditions

For closed cylindrical containers more than half-full of liquid, rotated about its vertical axis ($h > H/2$):



y/2 < D

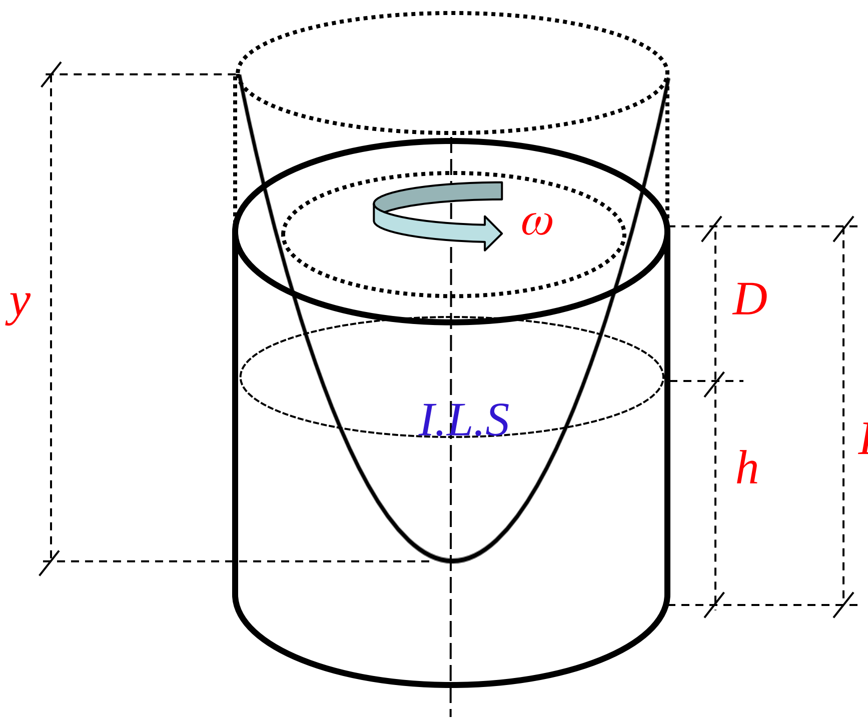


y/2 = D

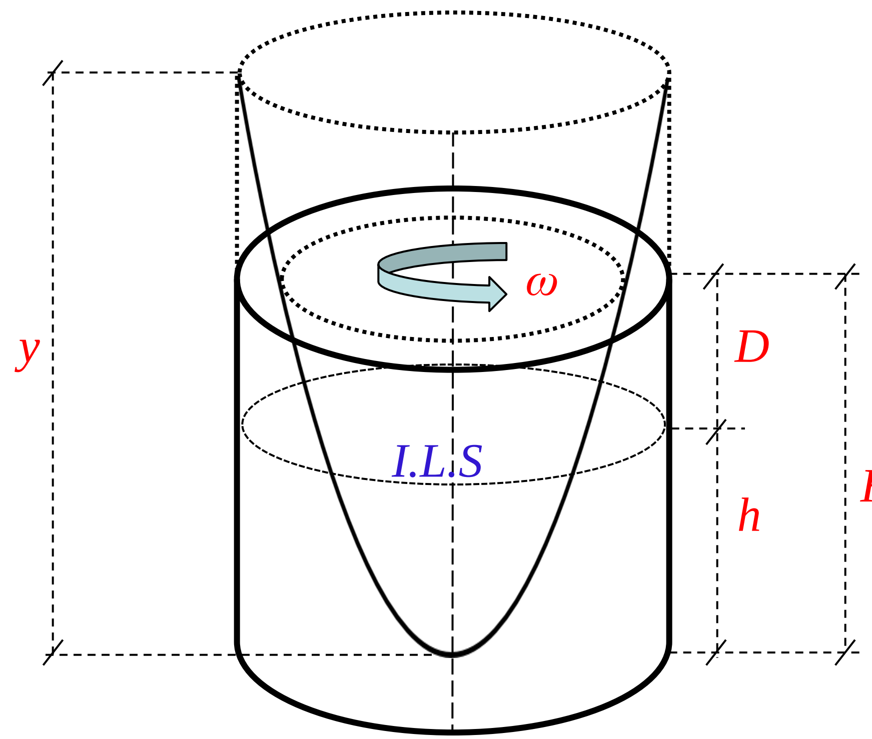
(Liquid surface just touching the top rim)

Relative Equilibrium of Liquids

Rotation (Rotating Vessel)



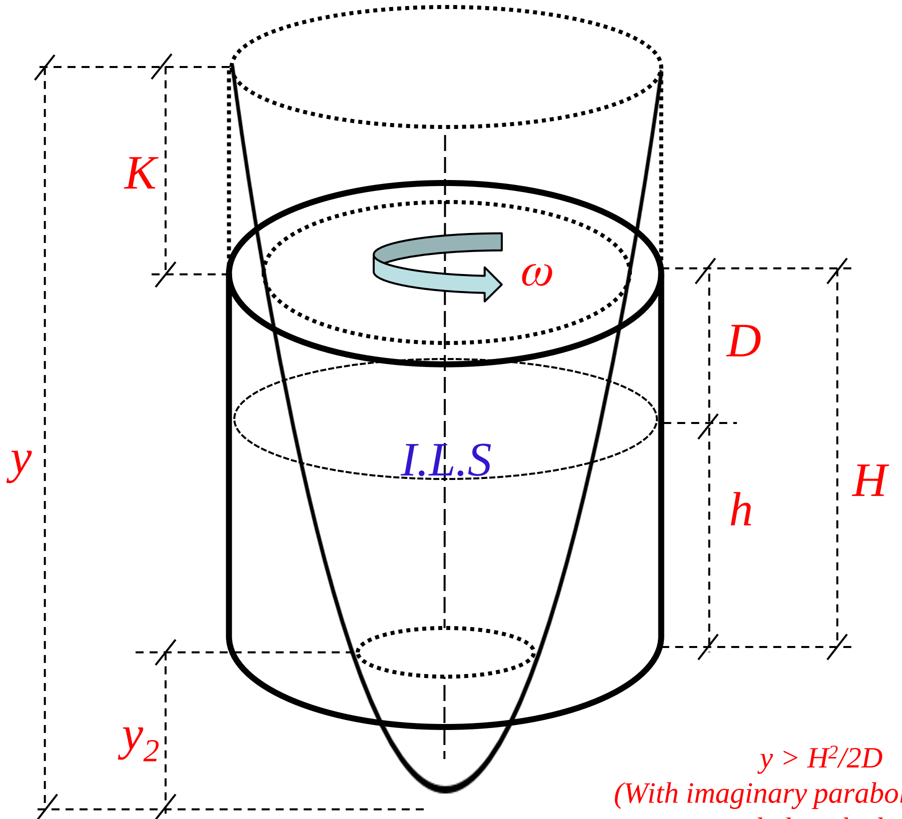
$y/2 > D$
(With imaginary paraboloid above)



$y = H^2/2D$
(With imaginary paraboloid above, vortex just touching the bottom)

Relative Equilibrium of Liquids

Rotation (Rotating Vessel)



$y > H^2/2D$
(With imaginary paraboloid above, vortex below the bottom)

Note: For closed vessels, there can never be any liquid spilled, so the initial volume of liquid (before rotation) is always equal to the final volume of the liquid (after rotation) or the initial volume of air inside is equal to the final volume of air inside. The volume of air relation is more convenient to use in solving this type of problem.

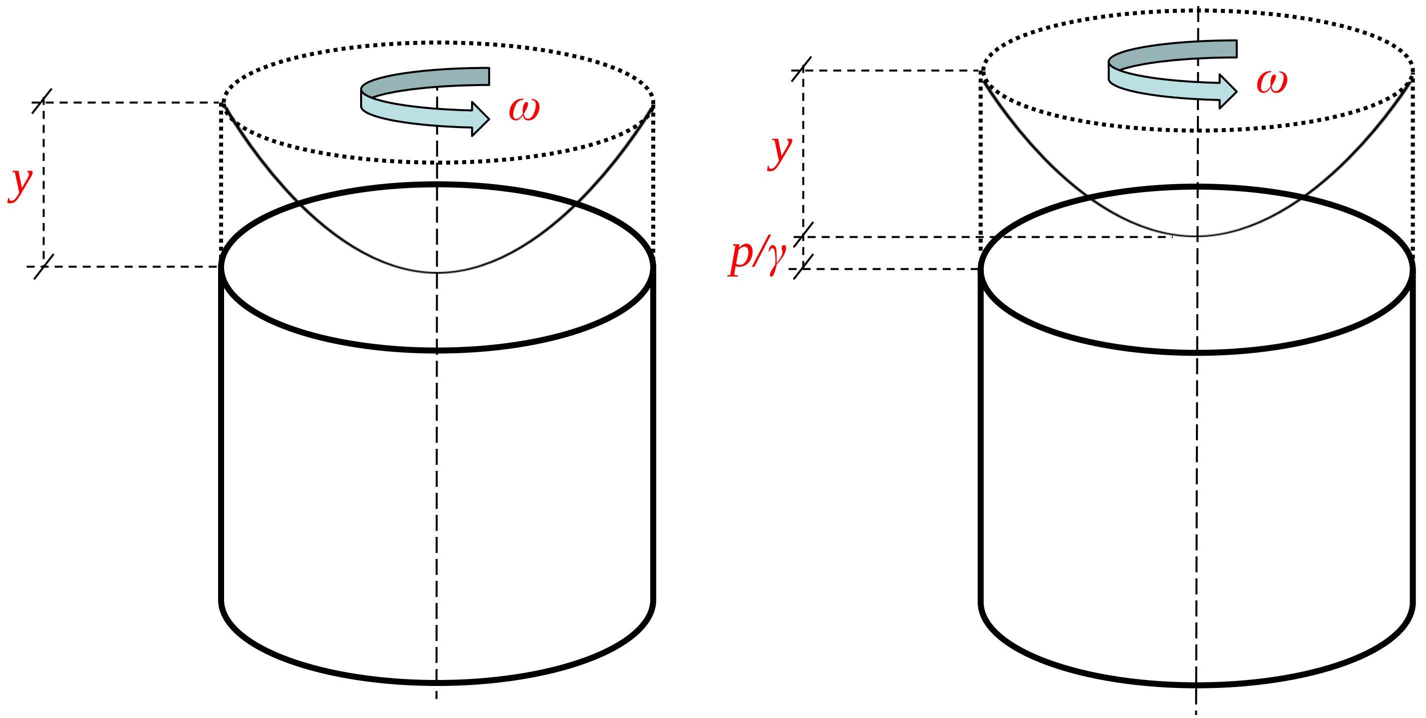
$$y_2 = (D/H)(y - K); \; K = H^2/2D$$

Relative Equilibrium of Liquids

Rotation (Rotating Vessel)

Liquid Surface Conditions

For closed cylindrical containers completely filled with liquid:



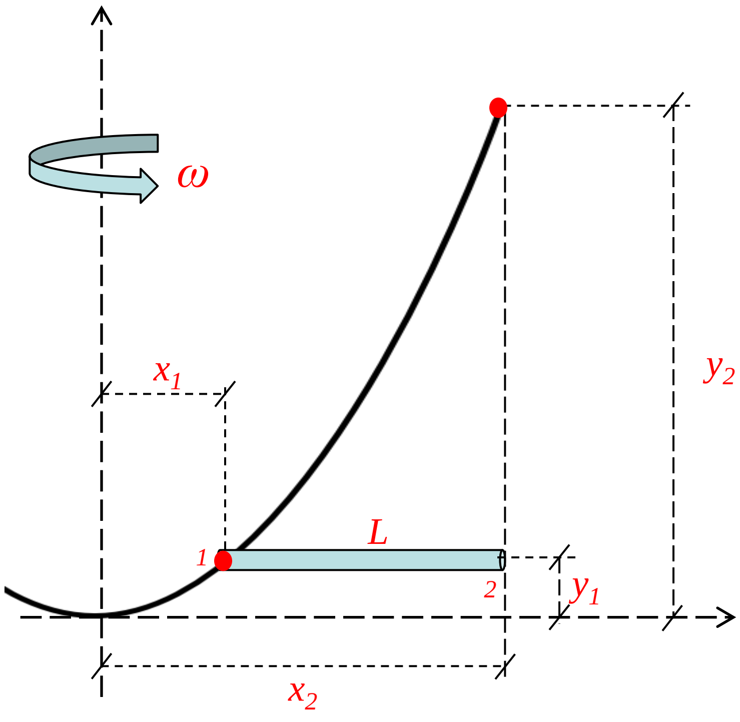
(Without pressure at top)

(With pressure at top)

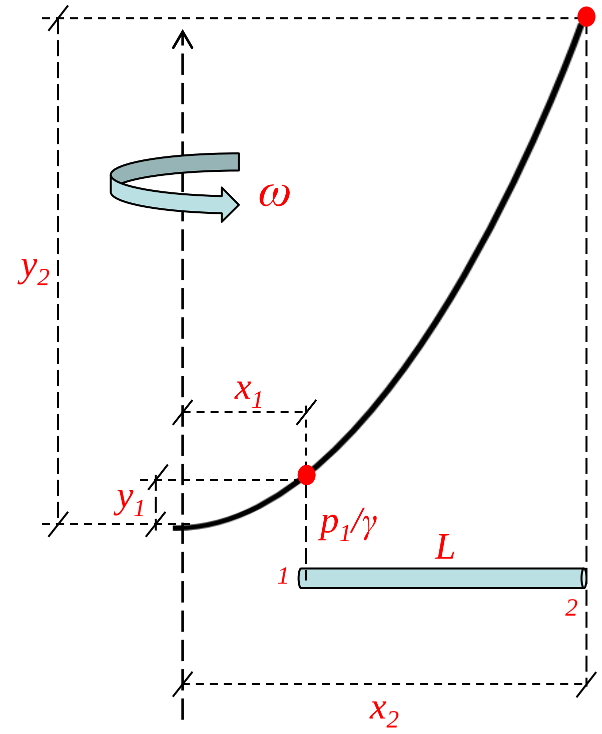
Relative Equilibrium of Liquids

Rotation (Rotating Vessel)

For pipes and tubes:



(Without initial pressure inside)

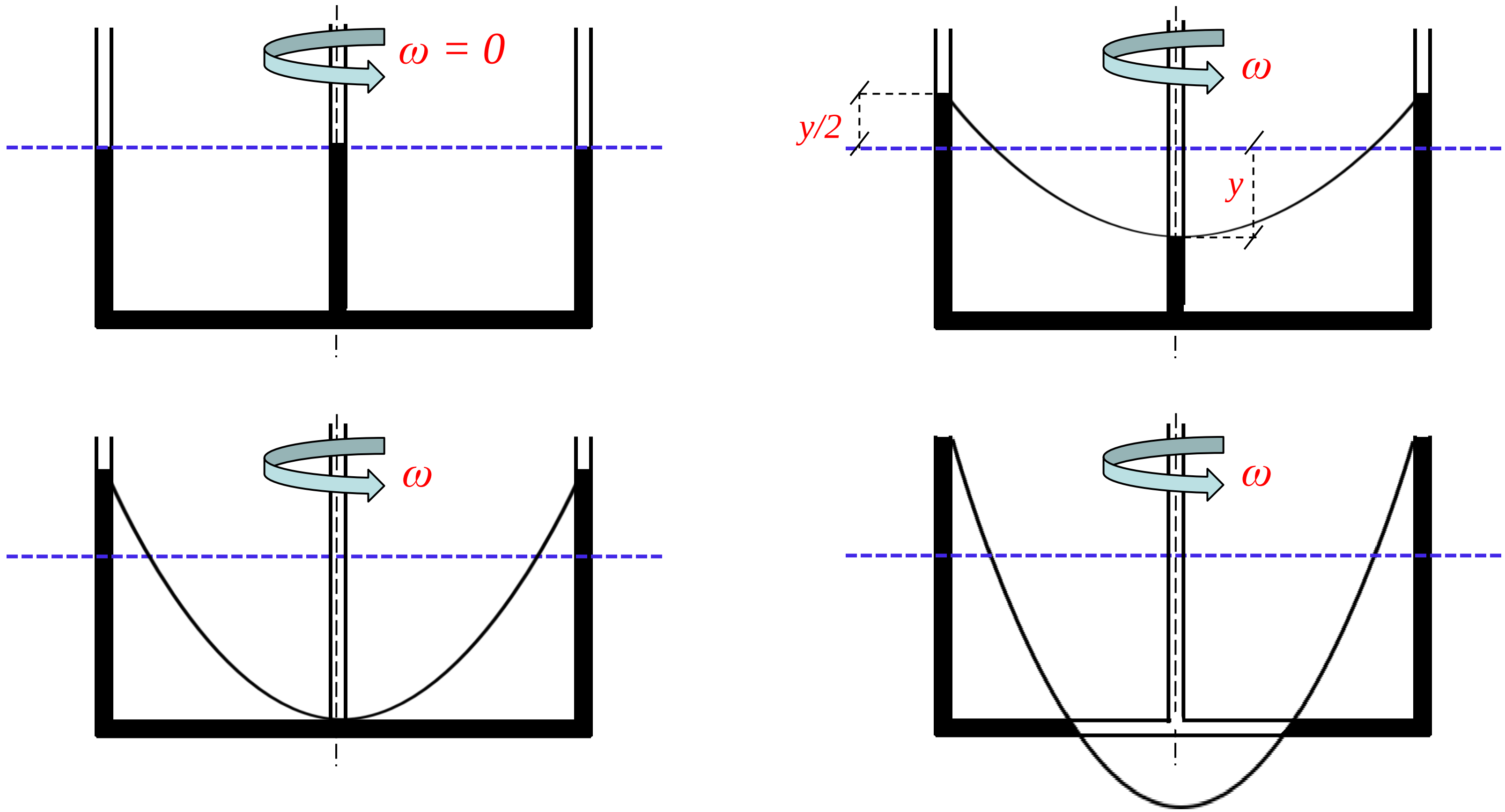


(With initial pressure inside)

Relative Equilibrium of Liquids

Rotation (Rotating Vessel)

U-tube revolved about its axis:



Relative Equilibrium of Liquids

Problem Set 11

Problem 1

An open cylindrical tank, 2 *m* in diameter and 4 *m* high contains water to a depth of 3 *m*. It is rotated about its own vertical axis with a constant angular speed, ω.

1.1 If ω = 3 *rad/s*, is there any liquid spilled. (*No liquid is spilled out*)

1.2 What maximum value of ω (*in rpm*) can be imposed without spilling any liquid?(ω = 59.78 *rpm*)

1.3 If ω = 8 *rad/s*, how much water is spilled out and to what depth will the water stand when brought to rest? ($V_{spilled}$ = 1.979 m^3)

1.4 What angular speed ω (*in rpm*) will just zero the depth of water at the center of the tank? (ω = 84.6 *rpm*)

1.5 If ω = 100 *rpm*, how much area at the bottom of the tank is uncovered? ($A_{uncovered}$ = 0.889 m^3)

Relative Equilibrium of Liquids

Problem Set 11

Problem 2

A closed cylindrical vessel, 2 *m* in diameter and 4 *m* high is filled with water to a depth of 3 *m* and rotated about its own vertical axis at a constant angular speed, ω. The air inside the vessel is under a pressure of 120 *kPa*.

- *2.1* If ω = 12 *rad/s*, what is the pressure at the center and circumference at the bottom of the tank? (p_{center} = 121.66 *kPa*, $p_{circumference}$ = 193.67 *kPa*)
- *2.2* What angular speed ω (*in rpm*) will just zero the depth of water at the center?(ω = 119.6 *rpm*)
- *2.3* If ω = 20 *rad/s*, how much area at the bottom is uncovered? (A = 0.48 m^2)

Relative Equilibrium of Liquids

Problem Set 11

Problem 3

A 75 *mm* diameter pipe, 2 *m* long is just filled with oil (s = 0.822) and then capped, and placed on a horizontal position. It is rotated at 27.5 *rad/s* about a vertical axis 0.5 *m* from one end (out the pipe). What is the pressure in *kPa* at the far end of the pipe?

Ans: p_{far} = 1,865 *kPa*

Relative Equilibrium of Liquids

Problem Set 11

Problem 4

A glass U-tube whose vertical stems are 600 mm apart is filled with mercury to a depth of 200 mm in the vertical stems. It is rotated about a vertical axis through its horizontal base 400 mm from one stem. How fast should it be rotated so that the difference in the mercury levels in the stems is 200 mm?

Ans: ω = 54.6 rpm

Relative Equilibrium of Liquids

Problem Set 11

Problem 5

A glass tubing consist of 5 vertical stems which are 500 *mm* apart connected to a single horizontal tube. The tube is filled with water to a depth of 500 *mm* in the vertical stems. How fast should it be rotated about an axis through the middle stem to just zero the depth of water in that stem?

Ans: ω = 42.3 *rpm*